

CH301 Fall 2009 Worksheet 2 Answer Key

1. Calculate wavelength from the following frequencies.

a) 625 kHz

$$\lambda = c/v$$

$$\lambda = (3.00 \times 10^8 \text{ m/s}) / (625 \times 10^3 \text{ Hz}) = 480. \text{ m}$$

b) 734 MHz

$$\lambda = (3.00 \times 10^8 \text{ m/s}) / (734 \times 10^6 \text{ Hz}) = .409 \text{ m}$$

c) 8.4×10^{14} Hz

$$\lambda = (3.00 \times 10^8 \text{ m/s}) / (8.4 \times 10^{14} \text{ Hz}) = 3.6 \times 10^{-7} \text{ m}$$

d) 92 GHz

$$\lambda = (3.00 \times 10^8 \text{ m/s}) / (92 \times 10^9 \text{ Hz}) = 3.3 \times 10^{-3}$$

2. Given the following energies, calculate the frequencies of the photons.

a) 17 kJ

$$v = E/h$$

$$v = 17000 \text{ J} / 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 2.6 \times 10^{37} \text{ s}^{-1}$$

b) 564×10^{-25} J

$$v = 564 \times 10^{-25} \text{ J} / 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 8.51 \times 10^9 \text{ Hz}$$

c) 98 pJ

$$v = 98 \times 10^{-12} \text{ J} / 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 1.5 \times 10^{23} \text{ Hz}$$

d) 230 J

$$v = 230 \text{ J} / 6.63 \times 10^{-34} \text{ J}\cdot\text{s} = 3.5 \times 10^{35} \text{ s}^{-1}$$

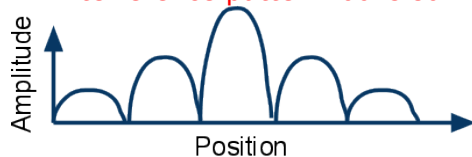
3. Rank the wavelengths of the photons in question 2 from longest to shortest. Use A, B, C, or D to identify the photon. (Hint: No calculations necessary.)

B C D A

Remember that wavelength and energy are inversely related ($E = h\cdot c/\lambda$) – the higher the energy, the shorter the wavelength.

4. The double slit experiment is a famous demonstration of the wave nature of light: when light of a single frequency is passed through two parallel slits in a barrier, it produces an *interference pattern* on a surface placed on the other side. Draw an example of an interference pattern (as amplitude of light received vs. position). Compare the frequency and phase of the light waves coming from the two slits at the dark spots and at the light spots on the interference pattern.

An interference pattern looks something like this:



At the dark spots, the two waves still have the same frequency as the original light source, but have opposite phase causing them to cancel (destructive interference). At the light spots, the frequency is also the same but the phase is identical, causing constructive interference.

5. When passed through very fine gratings (or even regular crystals), electrons and neutrons form a distinctive pattern of ridges and troughs because of their wave nature. What is this effect called?

The scattering of waves by lattices (the fancy name for a grating) is called diffraction. Fun fact: X-rays were first identified as light waves in the 19th century by the fact that they were diffracted by crystals. Knowing what we do now about the wave nature of matter,

would we necessarily draw the same conclusion from that evidence?

6. The description of light as a collection of particles goes back to long before quantum mechanics. Who are two examples of people who proposed such a description before 1800? The Greek philosopher Democritus proposed that light was composed of particles in the 5th century B.C (!), and Sir Isaac Newton gave a "Corpuscular Theory of Light" at the end of the 17th century.

7. Classical physics assumed that the intensity of radiation emitted was a function of λ and that at a given temperature as λ increases the intensity of radiation decreased. Therefore, since UV radiation has a short wavelength its intensity distribution should be large. Describe how Planck solved the ultraviolet catastrophe.

Planck proposed a radical idea, that the exchange of energy between matter and radiation occurs in quanta. A radiation with a certain frequency can be generated only if an oscillator (electron, atom, or molecule) of that frequency has acquired the minimum energy, $E = h\nu$ to start the oscillation. At low temperatures there is not enough energy to cause high frequency oscillations, hence the intensity of the curve is zero. A peak of intensity is reached at a certain wavelength and the intensity decreases again because there's not enough energy to stimulate higher frequency oscillators. No high energy radiation is emitted and the UV catastrophe is no more!

8. Before Einstein won the Nobel Prize for discovering the photoelectric effect, classical physics suggested that there is no minima of energy needed to eject electrons from a metal. Explain the principles of the photoelectric effect and how Einstein concluded that light is particle-like.

Classical physics failed because there's no relation to energy of incident light and that of the ejected electrons. Einstein explained that electromagnetic radiation consists of particles (later called photons) and that each photon is a quanta of energy related to the wavelength of its radiation ($E = h\nu$ or $E = hc/\lambda$). If the energy of the incident photon is less than the work function of the metal no electron will be ejected no matter how long you sit there and shine radiation on it. If the energy of the incident light is higher than the work function than the electron is ejected with a certain kinetic energy. These observations are expressed as: $KE = \frac{1}{2}mv^2 = h\nu - \Phi$. Einstein's work backed Planck's theory that energy exchange occurs in quanta.

9. What is the deBroglie wave equation? In your own words, what is its significance?

The deBroglie wave equation is $\lambda = h/(m*v) = h/p$. Matter, like EMR, has a wave particle duality, such that wavelength and mass have an inverse relationship.

10. Use the deBroglie equation to calculate the following values.

a) the wavelength of a 70.0 kg person traveling at 6.0 mi/hr.

$$\lambda = h/(m*v)$$

$$v = 6.0 \text{ mi/hr} * 1600 \text{ m/mi} * 1 \text{ hr}/3600 \text{ s} = 2.67 \text{ m/s}$$

$$\lambda = 6.63 \text{ E-34 J}\cdot\text{s} / (70 \text{ kg} * 2.67 \text{ m/s}) = 3.6 \text{ E-36 m}$$

b) the velocity of an e^- (mass 9.1 E-31 kg) with a wavelength of 450 pm.

$$v = h/(m*\lambda)$$

$$v = 6.63 \text{ E-34 J}\cdot\text{s} / (9.1 \text{ E-31 kg} * 450 \text{ E-12 m}) = 1.6 \text{ E6 m/s}$$

c) the mass of a particle traveling at the speed of light with a wavelength of 12 nm.

$$m = h/(v*\lambda)$$

$$m = 6.63 \text{ E-34 J}\cdot\text{s} / (3 \text{ E8 m/s} * 12 \text{ E-9 m}) = 1.8 \text{ E-34 kg}$$

11. What is the energy difference between the $n = 1$ and $n = 2$ energy levels for an electron

in a 1 Å box? What is wavelength of a photon with this energy?

$$\begin{aligned} E_2 - E_1 &= (2^2 - 1^2) \cdot h^2 / 8 \cdot m \cdot L^2 \\ &= 3 \cdot (6.6 \cdot 10^{-34} \text{ J}\cdot\text{s})^2 / 8 \cdot (9.1 \cdot 10^{-31} \text{ kg}) \cdot (1 \cdot 10^{-10} \text{ m})^2 \\ &= 1.8 \cdot 10^{-17} \text{ J} \end{aligned}$$

This corresponds to a photon of wavelength 11 nm, which is in the X-ray region.

12. Could the particle in a box solution be used to approximate atomic energy levels? Why or why not?

Since the energy levels of the particle in a box grow increasingly further apart as n increases while atomic energy levels grow increasing closer together, the particle in a box could not be used as a valid approximation.

13. One of the conditions at which quantum mechanics simplifies to classical mechanics is when the quantum number, n , approaches infinity. What is the probability density for the particle, $\Psi(x)^2$, for the particle in a box in this limit? Does this agree with the classical prediction?

As n increases, the peaks and troughs in $\Psi(x)^2$ get closer and closer together. At n equals infinity, these peaks merge into a flat line, $\Psi(x)^2 = 1/L$ (so that the total probability in the box is equal to 1). Since a classical particle simply bounces from one edge of the box to the other, the probability of finding it at any point in the box is the same as all other points in the box, which agrees with the n equals infinity limit.

14. Estimate the minimum uncertainty in the position of a stone with mass 1.0 g that is rolling at $\pm 1.0 \text{ mm}\cdot\text{s}^{-1}$?

$$\Delta p \Delta x \geq \hbar/2$$

$$\Delta x \geq \hbar / (2\Delta p) \quad p = m\Delta v$$

$$\Delta x \geq \hbar / (2m\Delta v)$$

$$\Delta x \geq 1.05457 \cdot 10^{-34} \text{ J}\cdot\text{s} / (2 \cdot 1 \cdot 10^{-3} \text{ kg} \cdot 2 \cdot 10^{-3} \text{ m}\cdot\text{s}^{-1})$$

$$\Delta x = 2.6 \cdot 10^{-29} \text{ m}$$

15. Estimate the minimum uncertainty in the speed of an electron of mass $9.109 \cdot 10^{-31} \text{ kg}$ if it is confined in the diameter of an atom (193 pm).

$$\Delta p \Delta x \geq \hbar/2$$

$$\Delta p \geq \hbar / (2\Delta x) \quad p = m\Delta v$$

$$m\Delta v \geq \hbar / (2\Delta x)$$

$$\Delta v \geq \hbar / (2m\Delta x)$$

$$\Delta v \geq 1.05457 \cdot 10^{-34} \text{ J}\cdot\text{s} / (2 \cdot 9.109 \cdot 10^{-31} \text{ kg} \cdot 1.93 \cdot 10^{-10} \text{ m})$$

$$\Delta v = 3 \cdot 10^5 \text{ m/s}$$

16. Give a brief explanation of each of the following quantum numbers in your own words:
 n, ℓ, m_ℓ, m_s .

n = principal quantum number, the main energy levels, any positive integer.

ℓ = azimuthal quantum number, shape of orbits, 0 to $n-1$.

m = magnetic quantum number, orientation of orbital, $-l$ to l

s = spin quantum number, direction of spin, $+1/2$ or $-1/2$ (NOTE: The value for this number does not matter, just that there is a positive and negative value of equal magnitudes).

17. True or False? The following set of quantum numbers is acceptable (0,0,0, $+1/2$). Explain.

False. The principal quantum number, n , corresponds to the volume of space in which an electron moves around a nucleus and can only be positive integers. Therefore, 0 is not a principle quantum number.

18. How many electrons can have the following quantum numbers in an atom: $n=3, l=2$?
10 electrons.

For every subshell (l) there are $-l$ to l possible orbitals, each of which may contain two electrons with opposite spins.

19. In the simplest way possible, define the Aufbau principle, the Pauli exclusion principle, and Hund's rule.

Aufbau principle: electron orbits are filled from lowest energy to highest energy.

Pauli exclusion principle: only 2 electrons per orbital

Hund's rule: electrons will distribute between orbitals of the same energy (example: If there are 3 electrons and three orbitals, there will be one electron in each orbit instead of one empty orbit, one orbit with one electron, and one orbit with 2 electrons.)

20. Fix the electron configurations and state which principle(s) is/are being violated.

a) Oxygen

2p	$\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow$	2p	$_ _ _$
2s	$_$	2s	$_$
1s	$\uparrow\downarrow$	1s	$_$

The Aufbau principle is being violated

2p	$\uparrow\downarrow \uparrow \uparrow$
2s	$\uparrow\downarrow$
1s	$\uparrow\downarrow$

b) Nitrogen

2p	$\uparrow\downarrow \uparrow _$	2p	$_ _ _$
2s	$\uparrow\downarrow$	2s	$_$
1s	$\uparrow\downarrow$	1s	$_$

The Hund's rule is being violated.

2p	$\uparrow \uparrow \uparrow$
2s	$\uparrow\downarrow$
1s	$\uparrow\downarrow$

c) Sulfur

3p	$\uparrow\downarrow\uparrow \uparrow\downarrow\uparrow _$	3p	$_ _ _$
3s	$\uparrow\downarrow$	3s	$_$
2p	$\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow$	2p	$_ _ _$
2s	$_$	2s	$_$
1s	$\uparrow\downarrow$	1s	$_$

The Aufbau principle, the Pauli exclusion principle, and Hund's rule are being violated.

3p	$\uparrow\downarrow \uparrow \uparrow$
3s	$\uparrow\downarrow$
2p	$\uparrow\downarrow \uparrow\downarrow \uparrow\downarrow$
2s	$\uparrow\downarrow$
1s	$\uparrow\downarrow$